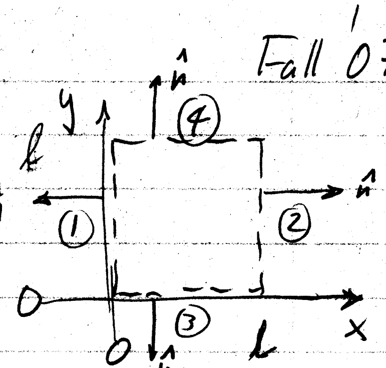
QE Problem F06 Fall 07



$u = y$, $v = x$

a) $I = I_1 + I_2 + I_3 + I_4$, $I = \int \rho \vec{V} \cdot \hat{n}\, dA$

On ①, $\hat{n} = -\hat{i}$, $\vec{V} = y\hat{i} + 0\hat{j}$ since $x = 0$

$dA = dy$, $\vec{V} \cdot \hat{n} = -y$, $I_1 = \int_0^{\ell} -\rho y\, dy = -\frac{1}{2}\rho \ell^2$

On ②, $\hat{n} = \hat{i}$, $\vec{V} = y\hat{i} + \ell\hat{j}$ since $x = \ell$

$dA = dy$, $\vec{V} \cdot \hat{n} = y$, $I_2 = \int_0^{\ell} \rho y\, dy = \frac{1}{2}\rho \ell^2$

On ③, $\hat{n} = -\hat{j}$, $\vec{V} = 0\hat{i} + x\hat{j}$ since $y = 0$

$dA = dx$, $\vec{V} \cdot \hat{n} = -x$, $I_3 = \int_0^{\ell} -\rho x\, dx = -\frac{1}{2}\rho \ell^2$

On ④, $\hat{n} = \hat{j}$, $\vec{V} = \ell\hat{i} + x\hat{j}$ since $y = \ell$

$dA = dx$, $\vec{V} \cdot \hat{n} = x$, $I_4 = \int_0^{\ell} \rho x\, dx = \frac{1}{2}\rho \ell^2$

$$\boxed{I = I_1 + I_2 + I_3 + I_4 = -\tfrac{1}{2}\rho\ell^2 + \tfrac{1}{2}\rho\ell^2 - \tfrac{1}{2}\rho\ell^2 + \tfrac{1}{2}\rho\ell^2 = 0}$$

b) To satisfy mass conservation at each point x, y, we must have $\nabla \cdot \vec{V} = 0$, for low speed flow (as given)

Check: $\boxed{\nabla \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial (y)}{\partial x} + \frac{\partial (x)}{\partial y} = 0 + 0 = 0}$ Yep.